

Course Code: 4MSCM1  
 Course: Functional Analysis-II  
 Credit: 4  
 Last Submission Date: October 31, (for January session)  
 April 30 (for July Session)

Max. Marks:-70  
 Min. Marks:-25

Note:-attempt all questions.

- Que.1 State & prove Closed graph theorem.
- Que.2 State & prove Hahn – Banach theorem for linear space.
- Que.3 If  $X$  is an inner product space and  $x, y \in X$ . Then
- $$| \langle x, y \rangle | \leq \|x\| \cdot \|y\|$$
- Que.4 Use Cauchy's inequality to prove Schwarz's inequality in the Hilbert space  $l_2^n$ .
- Que.5 Let  $\{e_1, e_2, \dots, e_n\}$  be a finite orthonormal set in a Hilbert space  $H$  and  $x$  be any element to  $H$ . Then  $\sum_{i=1}^n |\langle x, e_i \rangle|^2 \leq \|x\|^2$
- Que.6 State & prove Projection theorem.
- Que.7 State & prove Riesz Representation theorem for continuous linear function on a Hilbert space.
- Que.8 Every Hilbert space  $H$  is reflexive.
- Que.9 Let  $T$  be an operator on  $H$  define the adjoint  $T^*$  of  $T$ . the mapping  $T \rightarrow T^*$  of  $B(H)$  into itself has the following properties, for  $T, T_1, T_2 \in B(H)$  and  $\alpha \in \mathbb{C}$  we have
- (1)  $(T_1 + T_2)^* = T_1^* + T_2^*$
  - (2)  $(\alpha T)^* = \bar{\alpha} T^*$
  - (3)  $(T_1 T_2)^* = T_2^* T_1^*$
  - (4)  $T^{**} = (T^*)^* = T$
  - (5)  $\|T^*\| = \|T\|$
  - (6)  $\|T^* T\| = \|T\|^2$
- Que.10 If  $T_1$  and  $T_2$  are normal operators on a Hilbert space  $H$  with the property that either commutes with the adjoint of the other, then  $T_1 + T_2$  and  $T_1 T_2$  are normal.