Course Code: 2MSCM2
Course: Real Analysis-II
Credit: 4
Last Submission Date: October 31, (for January session)
April 30 (for July Session)

Max. Marks:-70
Min. Marks:-25

Note:-attempt all questions.

Que.1 Let f be a bounded function and $\propto$ be a monotonically increasing function on [a,b]. Then f ϵR ($\propto$) on [a,b] if and only if for every $\in > 0$ there exists a partition p such that

$$U(p,f , \propto) - L (p,f, \propto) < \in$$

Que.2 If $f_1 \in R(\propto)$ and $f_2 \in R(\propto)$ on [a , b] then

$$f_1 + f_2 \in R(\propto) \text{ and } \int_a^b (f_1 + f_2)\, d\propto = \int_a^b f_1 \, d\propto + \int_a^b f_2 \, d\propto$$

Que.3 Let $\propto$ be a monotonically inversing function on [a, b] and $\propto' \in R[a, b]$. Let f be a bounded real function on [a, b]. Then $f \in R(\propto)$ if and only if $f\propto' \in R[a,b]$. In that case $\int_a^b f d\propto = \int_a^b f(x) \propto'(x) dx$

Que.4 Let f(x) = x and $\propto(x) = x^2$. Does $\int_0^1 f d\propto$ exist? If it exists, find its value.

Que.5 The sum function of a uniformly convergent series of continuous functions is itself continuous.

Que.6 Suppose $\{ f_n \}$ is a sequence of function, differentiable on [a,b] and such that $\{ f_{n(x0)} \}$ converges for some point x_0 on [a,b] . If $\{ f'_n \}$ converges uniformly on [a,b]. Then $\{ f_n \}$ converges uniformly on [a,b] to a function f , and $f'(x) = \lim_{n\to\infty} f'_n(x)$ ($a \le x \le b$)

Que.7 State and prove Implicit function theorem.

Que.8 If $f(t) = \int_{-\infty}^{\infty} e^{-x^2} . \cos(xt)dx$

And $g(t) = -\int_{-\infty}^{\infty} xe^{-x^2} . \sin(xt)dx$ for $-\infty < t < \infty$. Then prove that both integrals exist and f is differentiable and f'(t) = g (t)

Que.9 Prove that $\frac{\partial (u,v,w)}{\partial(x,y,z)} \times \frac{\partial (x,y,z)}{\partial(u,v,w)} = 1$ i.e. J J' =1

Que.10 If $u^3 + v^3 = x + y$ And $u^2 + v^2 = x^3 + y^3$. Then show that

$$J(u,v) = \frac{\partial (u,v)}{\partial (x,y)} = \frac{y^2 - x^2}{2uv(u-v)}$$