

Course Code: 4MSCM2

Course: Integral Transform-II

Credit: 4

Last Submission Date: October 31, (for January session)

April 30 (for July Session)

Max. Marks:-70

Min. Marks:-25

Note:-attempt all questions.

Que.1 Using the Fourier sine transform, solve the partial differential equation $\frac{\partial v}{\partial t} = k \frac{\partial^2 v}{\partial x^2}$ for $x > 0$, $t > 0$, under the boundary conditions $V = v = 0$ when $x = 0$, $t > 0$ and the initial condition $v = 0$, when $t = 0$, $x > 0$.

Que.2 Use finite Fourier transforms to solve

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}$$

$$U(0, t) = 0, U(4, t) = 0, U(x, 0) = 2x, \text{ where } 0 < x < 4, t < (\infty).$$

Que.3 Find the Hankel transform of $x^{-2} e^{-x}$ taking $x^{-1} J_1(px)$ as the kernel.

Que.4 Find $H^{-1}\{P^{-2} e^{-ap}\}$, taking $n = 1$.

Que.5 State & Prove Parseval's theorem.

Que.6 $H_n\left(\frac{df}{dx}\right) = \int_0^a \frac{df}{dx} J_n(px) dx$, where p is the root of the equation $J_n(pa) = 0$.

Que.7 Prove that the finite Hankel transform of $f(x)$, $0 \leq x \leq 1$ is $P^{n-m} J_n(p)$, where

$$f(x) = \frac{2^{1+n-m}}{\sqrt{(m-n)}} (x^n (1-x^2)^{m-n-1})$$

Que.8 Find the potential $V(r, z)$ of a field due to a flat circular disc of radius with the centre of the origin and axis along the $Z =$ axis, satisfying the differential equation

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} = 0, 0 \leq r < \infty, z \geq 0 \text{ and the boundary conditions}$$
$$v = v(0) \text{ when } z = 0, 0 \leq r \leq 1 \text{ and } \frac{\partial v}{\partial z} = 0 \text{ when } z = 0, r > 1.$$

Que.9 State & Prove Mellin Inversion theorem.

Que.10 Find $M\{\sin x\}$