Course Code: 4MSCM3
Course: Special Functions-I
Credit: 4
Last Submission Date: October 31, (for January session)
April 30 (for July Session)

Max. Marks:-70
Min. Marks:-25

Note:-Attempt all questions.

Q.1 Prove that Rodrigues formula for $H_n(x)$

i.e. for $H_n(x) = (-1)^n \, exp\,(x^2) \, \frac{d^n}{dx^n} \{exp\,(-x^2)\}$

Q.2 Prove that $2x\, H_n\,(x) = \, 2_n H_{n-1}\,(x) + H_{n+1}(x)$

Q.3 Prove that

1) $\int_{-\infty}^{\infty} exp\,(-x^2)\, H_n(x)\, H_m(x) dx = 0 \; if \quad m \neq n$

2) $\int_{-\infty}^{\infty} exp\,(-x^2)\, H_n(x)\, H_m(x) dx = 2^n . n! . \sqrt{\pi} \; if \quad m = n$

Q.4 Show that

$\int_{-\infty}^{\infty} x^2 \; e^{-x2} \; \{H_n(x)\}^2 \, dx = \sqrt{\pi} \, .2^n \, .n! \, (n + \frac{1}{2})$

Q.5 Prove that

$(n+1)L_{n+1}(x) = (2n + 1 - x)L_n(x) - xL_{n-1}(x)$

Q.6 Prove that

$\int_s^{\infty} e^{-y} \, L_n\,(y) dy = e^{-x} \, [L_n\,(x) - L_{n-1}\,(x)]$

Q.7 Show that

$H_{2n}(x) = (-1)^n . 2^{2n} \, .n! \; L_n^{-1/2}\,(x^2)$

Q.8 Show that

$L_n^{(\propto)}\,(x) = \frac{x^{-\propto}\, e^x}{n!} . D^n \, [e^{-x} . x^{n+\propto}]$

Q.9 Prove that

$P_n^{(\propto,\beta)}\,(x) = \frac{(x-1)^{-\propto}(x+1)^{-\beta}}{2^n . n!} \, D^n \left[(x-1)^{n+\propto} . (x+1)^{n+\beta}\right]$

Q.10 Prove that

$\int_{-1}^{1} (1-x)^{\propto} (1+x)^{\beta} \, x^m \, P_n^{(\propto,\beta)}(x) dx = 0$

When m=0 ,1 ,- - - - - - -, (n-1)